ACCELERATOR RESEARCH PROGRESS MEETING

14th February, 1958.

COLLIDING BEAM ACCELERATORS

1. Requirements for a synchrocrash beam

1.1 Energy relations

If two particles, each with total energy E_{sc} in the laboratory system, collide head-on, the energy available in the centre of mass system is $2E_{sc}$. The energy in the centre system for a particle, with energy E_L in the laboratory, hitting a stationary target particle is given by

$$E^2_{c.m.} = 2M_o^2 + 2M_oE_L$$

where M_o is the rest mass of the particles.

Thus to compare a synchrocrash accelerator and a single beam accelerator we have the relations

$$(2E_{sc})^2 = 2M_o^2 + 2M_oE_L$$

Putting in numbers we find that

15 GeV Synchrocrash ≡ 500 GeV with the target at rest.

7 GeV " ≡ 100 GeV " " " " "

1.2 Intensity and yield

The number of interactions per second

$$= 2N_1N_2 \nu l \sigma A$$ ~~≑ 2Alσ~~

$$= 2Al\sigma\left\{\frac{N}{(2\pi RA)}\right\}^2 c$$

where N_1, N_2 = No. of particles/sq. cm. in each beam.
A = cross section area of beam.
l = length of target section.
σ = cross section for interaction.
ν = c = velocity of particles, which is taken to be velocity of light.
N = Total no. of particles in each beam.

Putting σ equal to the geometrical cross-section of the proton

$$= 5 \times 10^{-26} \text{ cms}^2$$

and $N = 5 \times 10^{14}$

No. of interactions/sec. = 10^7 for A = 1 sq. cm.
l = 1 metre.
R = 100 metres.

If the distribution of reaction products is isotropic, this gives 100 disintegration products per sq. cm. at 1 metre distance from the target section.

1.3 Background

The number of target protons per c.c.

$$= \frac{N}{2\pi Rh\Delta R}$$

where beam is assumed to have a rectangular cross-section of height h and breadth ΔR. If there are n gas nucleons per c.c. then the 'background ratio' equals

$$\frac{n \times 2\pi R h \Delta R}{N}$$

At 10^{-5} mms pressure, $n = 5 \times 10^{12}$, and taking h = 1 cm, R = 100 metres we obtain

$$\text{Ratio} = \frac{3 \times 10^{17} \Delta R (\text{in cms})}{N}$$

For $N \sim 10^{15}$, pressures of 10^{-8} – 10^{-9} mms would be required.

1.4 Beam lifetime

The ratio of Coulomb cross-section to nuclear cross-section at 15 GeV is approximately $\frac{1}{6}$.

The rate of reaction equals $n v \sigma$. At 10^{-5} mms pressure, $n = \cdot 5 \times 10^{13}$ nucleons per c.c., thus

$$\text{rate of reaction} = 5 \times 10^{12} \times 3 \times 10^{10} \times 5 \times 10^{-26}$$

$$= 7.5 \times 10^{-3}/\text{sec.}$$

Thus the mean life of the beam against reaction is 130 secs. Allowing for Coulomb scattering this lifetime is reduced to 100 secs.

2. Maximum circulating charge

The essence of the problem is to obtain a concentrated stack, with a particle density of the order of 5×10^{14} particles per sq. cm. of cross section area of the beam. This is equivalent to 50 amps/sq.cm. circulating current.

The circulating charge at full energy can be written as the product of factors in three ways (assuming no loss of particles during acceleration).

(i) N = Number protons accelerated per sec. x the circulating time.

(ii) = No. of protons per buckets x No. of buckets per sec. x the circulating time.

(iii) = No. of protons per bucket x total No. of buckets.

If a current of 30 milliamps at $v = .3c$ is injected for one turn, this is equivalent to 100 milliamps per bucket at full energy ($v = c$). If 10^4 buckets are accumulated at full energy, the circulating current is given by (iii)

$$= 100 \text{mamps} \times 10^4$$

$$= 1,000 \text{ amps.}$$

This would give a factor 20 in hand. Written in form (ii) this might be

$$N = 10^{12} \times 10^2 \times 10^2$$

2.1 Number of protons injected per bucket

With single turn injection, the number of particles injected is given by

$$N = \frac{2\pi R}{e \beta c} I \quad \text{(I is current from the injector (Linac))}$$

$$= \frac{2\pi \times 100 \times 10^{-3}}{\beta \times 3 \times 10^8 \times 1.6 \times 10^{-19}}$$

$$= \frac{10^{10}}{\beta} / \text{milliamp for R = 100 metres.}$$

With a 30 milliamp injector and $\beta = .3$, 10^{12} particles are injected in one turn.

2.2 The number of particles accelerated per second

If 10^{14} particles are accelerated per second, the mean current is

$$I_{mean} = 10^{14} \times 1.6 \times 10^{-19} \times 10^{6} \ \mu \text{ amps}$$

$$= 16 \ \mu \text{ amps.}$$

The power required to accelerate this mean current to 15 GeV is

$$P = VI$$

$$= 16 \times 10^{-6} \times 15 \times 10^{9}$$

$$= 240 \text{ kilowatts.}$$

Thus with two beams (as in an Ohkawa machine) there would be $\frac{1}{2}$ megawatt beam loading. This beam loading cannot reasonably be increased by an order of magnitude.

Thus with 100 seconds circulating time, 10^{16} is the upper limit for the number of particles in each beam. This beam would be made up of 10^{4} buckets and in order to find the energy spread (and thus the radial spread) of this beam, the process of beam stacking must be examined.

3. Beam stacking

This system of acceleration seems to be the most feasible and is the only one that has been studied in detail.

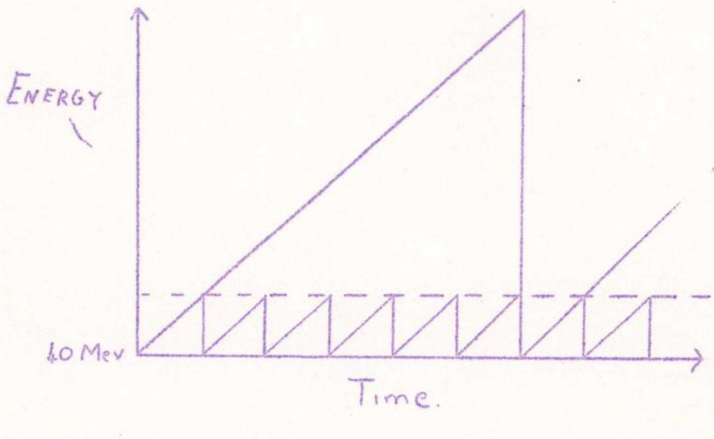

Fig. 1. Beam Stacking

3.1 Liouville's Theorem

The equation of motion for r-f acceleration is

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_0}\right) = \frac{d}{dt}\left(\frac{E_s}{K\,\omega_0^2}\,\dot{\phi}\right) = \frac{eV}{2\pi}\left(\sin\phi - \sin\phi_s\right)$$

Liouville's theorem tells us that if we plot $\frac{\Delta E}{\omega}$ against ϕ,

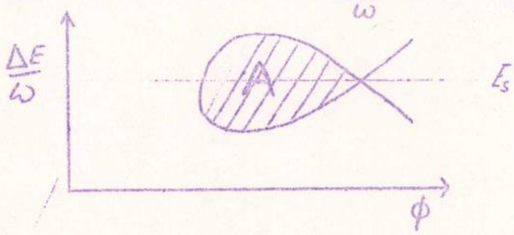

Fig. 2. Bucket Area

then phase area in these co-ordinates is conserved.

(1) This means that if we know the energy spread from the injector, then we know the energy spread at full energy.

(2) Because of 'phase displacement' if we take 'n' separate buckets up, we will get n times this energy spread.

3.2 Adiabatic switching

Knowing the energy spread from the injector, we may calculate as shown in 3.1 the energy spread of the beam at full energy. However, because this energy spread is somewhat of a debateable quantity, we prefer to estimate the phase area occupied by a bucket by considering what happens to a monoenergetic 'line' of particles when the r.f. voltage is switched on. This line will, of course, remain a line (by Liouville's theorem) and will thus have no area. However, because of non-linearities the line becomes 'twisted up' and can be thought of as enclosing an area. In order to keep this area small, when the particles are condensed into an accelerating bucket, the r-f voltage must be switched on adiabatically. We have considered a mode of switching, which is quick and also keeps the area small.

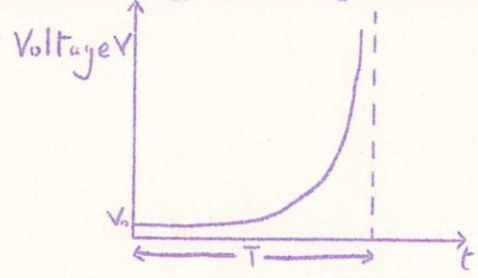

Fig. 3. Adiabatic Voltage Rise

This voltage rise law is

$$V = \frac{V_0 T^2}{(T - t)^2} \quad \text{where T is total time taken for switching.}$$

To make the phase area occupied by a pulse smaller, the switching time must be increased.

3.3 The first stage

The first stage of any stacking system is the most important and imposes the most severe limitations on the system. The first stacking level should be as low as possible to cut down acceleration time (e.g. 10 m.sec. requires 30 KeV/turn to go from 40 MeV to 100 MeV). The time taken for adiabatic switching must also be taken into account in calculating the duty cycle. Thus if a small energy spread is required for the final beam, the phase area per pulse must be small, and thus in turn the repetition rate in the first stage must be low. A balance must be obtained between the requirements of small energy spread and high repetition rate. These limitations are shown in Fig. 4 below.

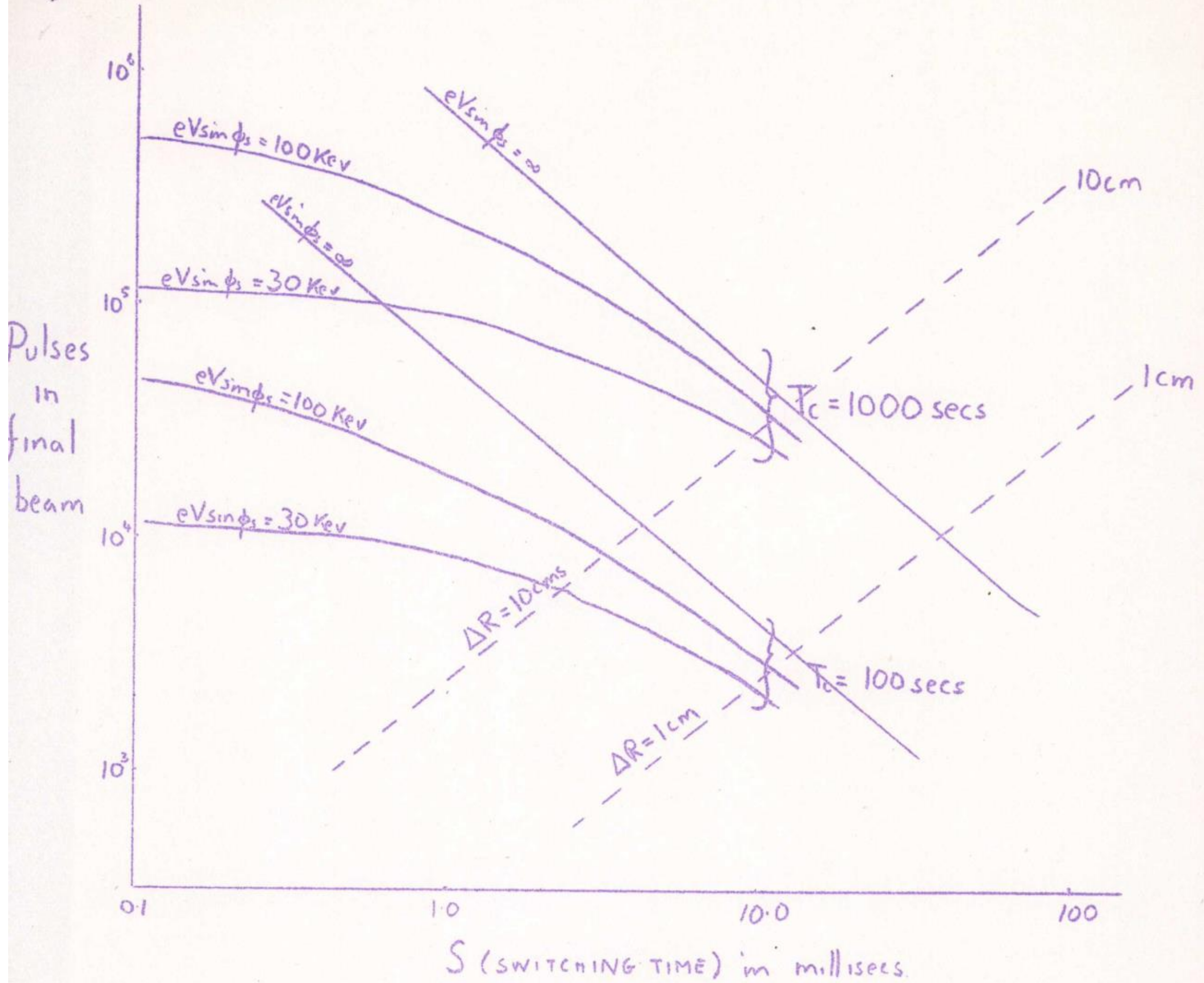

Fig. 4. Limitations in First Stage

With a switching time of 4 millisecs and a circulating time of 200 seconds, we obtain a final beam of 10^4 buckets with a radial spread of 10 cms.

3.4 Voltage tapering

The energy spread of the beam may be improved if we make the accelerating voltage vary with radius, and thus with energy. The phase equation then becomes

$$\frac{d}{dt}\left(\frac{\Delta E}{V\omega}\right) = \frac{e}{2\pi}\left(\sin\phi \quad - \sin\phi_s \qquad\right)$$

Liouville's theorem must now be applied with co-ordinates $\frac{\Delta E}{\omega V}$ and ϕ measuring the phase area. Thus if the voltage is dropped during acceleration, in order for area to be conserved, ΔE must also decrease.

Further numerical investigation of this process is needed and this will be done, when the 'Mercury computer' is working.

J. W. Burren

Building 412,
A.E.R.E., Harwell.
24th March, 1958.